**Bunbury Senior High School Name:\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Total time allowed: 55 minutes

Test 6 Total Marks \_\_\_\_\_ / 50 \_\_\_\_\_%

**Mathematics: Methods Unit 2**

**Test 6, 2018**

**Differentiation and Integration with Applications**

**Section One:**

**Calculator-free**

Time allowed for this section: 20 minutes

Total marks for this section: 18 marks

**Materials required for this section:**

SCSA Formula Sheet

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

|  |  |
| --- | --- |
| **Question 1** | **[1, 2 = 3 marks]** |

a) Differentiate the following functions. Express all derivatives with positive powers.

i)

✓ correct

ii)

✓ correctly applies to the negative power term

✓ correct with positive powers

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| **Question 2** | **[3, 3 = 6 marks]** |

a) Determine the equation of the tangent to the curve at the point (2, -11).

✓ correct derivative

✓ evaluates derivative at

solving

✓ correctly determines equation of the tangent

b) Determine the *x*-coordinates of any stationary point(s) of the function

✓ correct derivative

solving ✓ indicates stationary points occur at points of zero gradient

*x*-coordinates of the two stationary points are,

and ✓ correct *x*-coordinates

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| **Question 3** | **[2 marks]** |

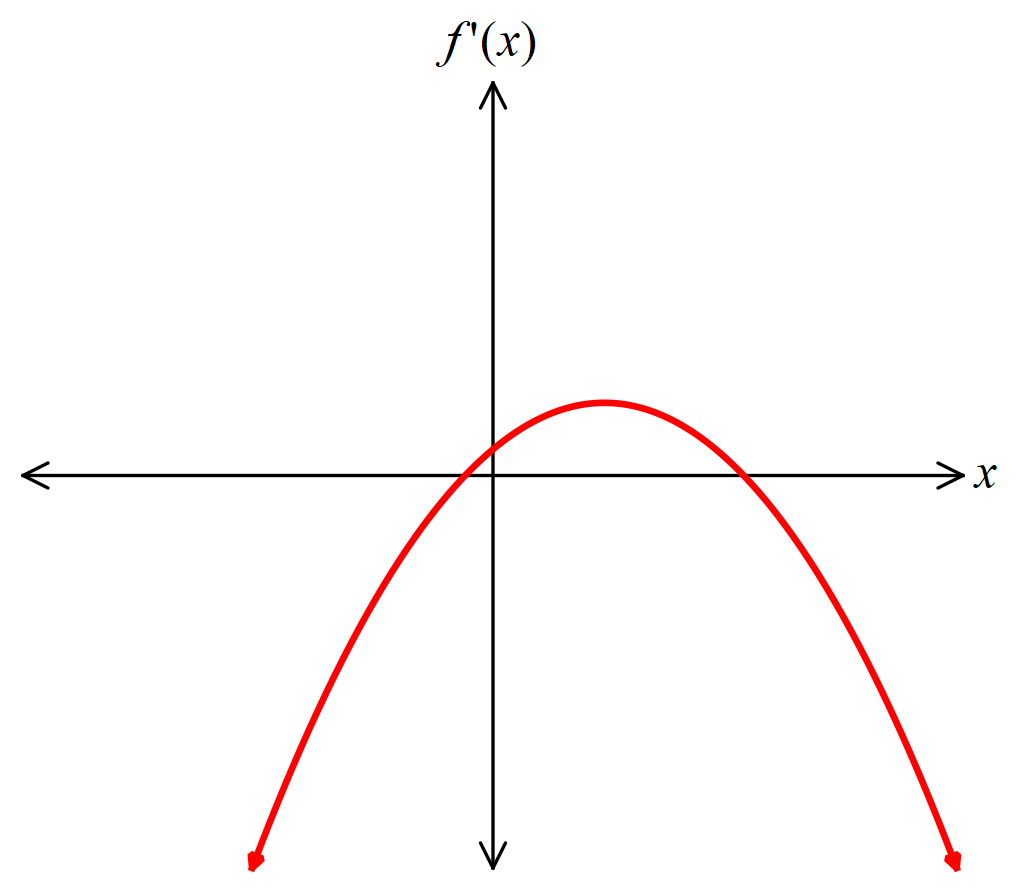
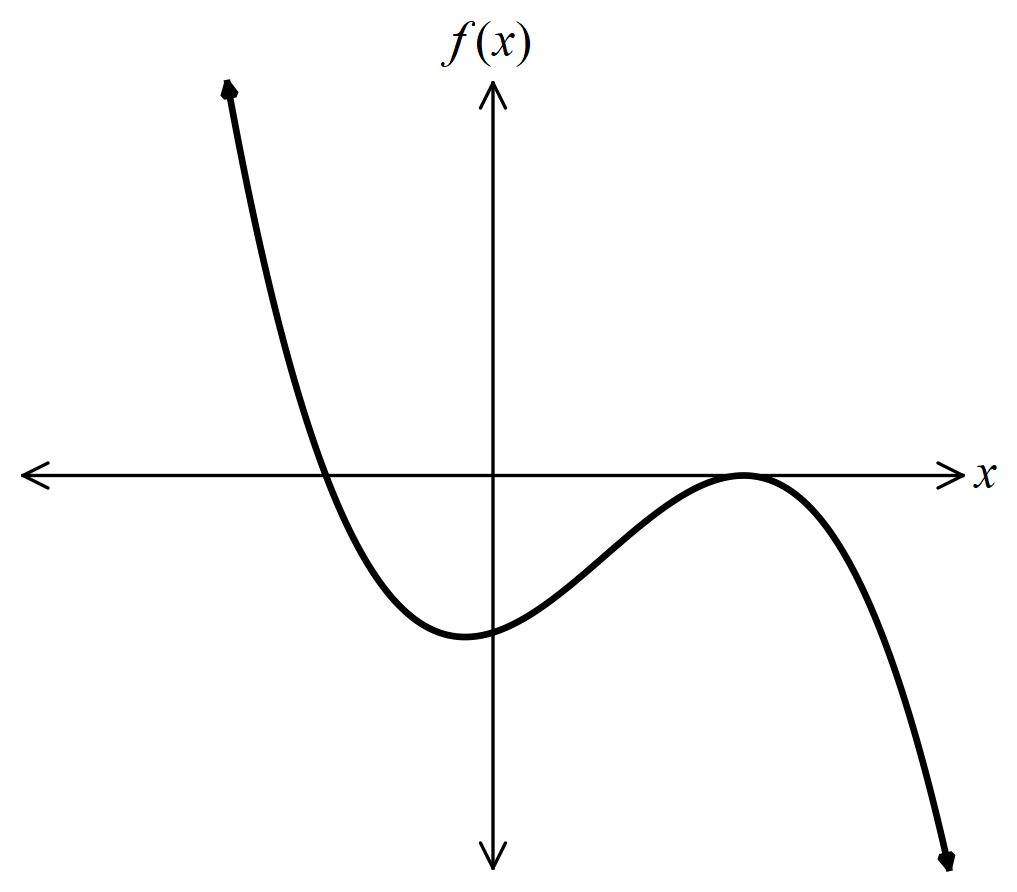
Determine the following integral.

✓ correctly applies anti-derivative to polynomial expression

✓ correctly simplifies with constant of integration included

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| **Question 4** | **[2 marks]** |

Given the sketch of , sketch a possible graph of on the axis below.

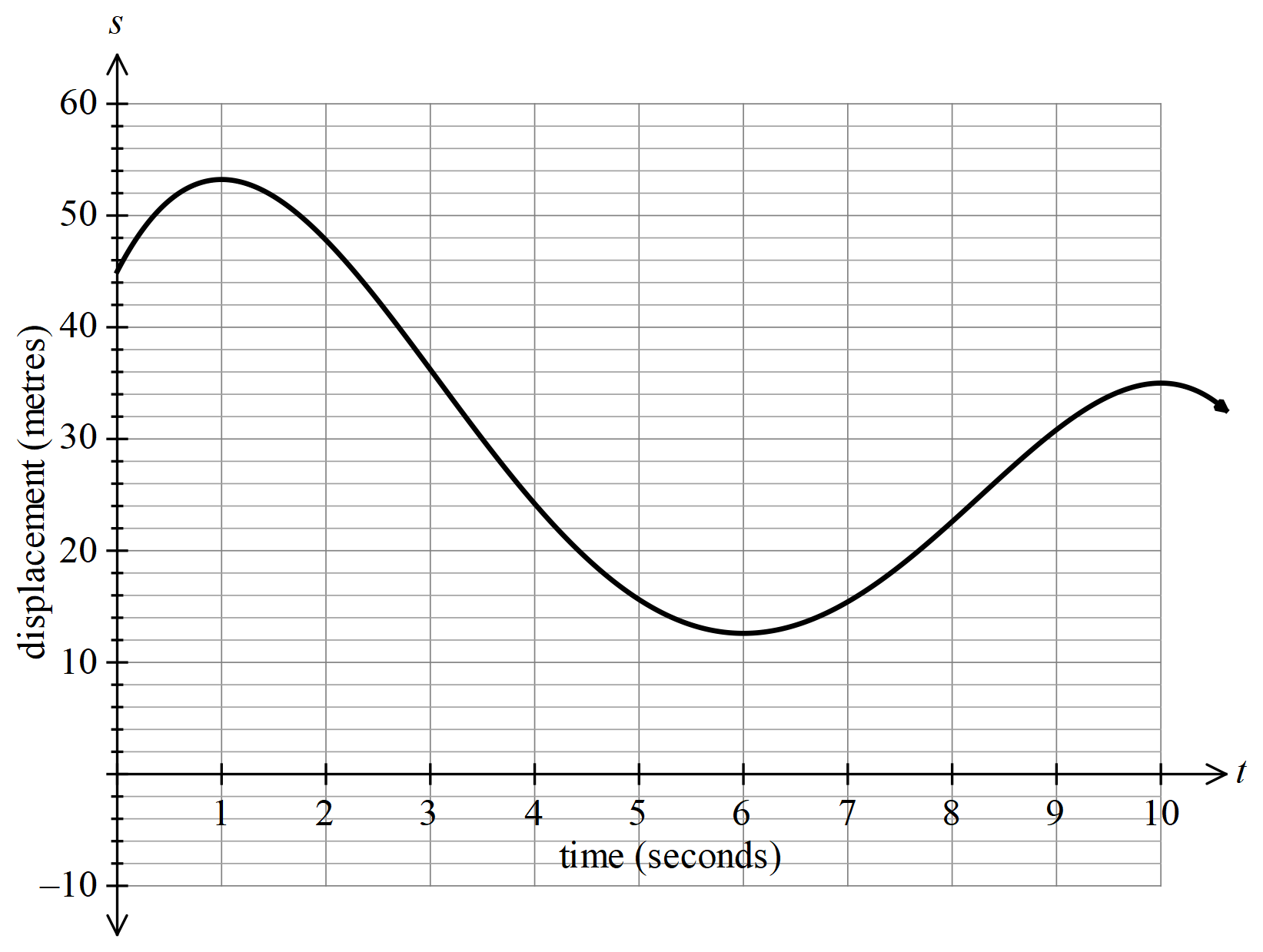


✓ shows roots of occur when has a turning point.

✓ correct parabola shape.

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| **Question 5** | **[1, 2, 1, 1 = 5 marks]** |

The accompanying diagram shows the displacement (metres) time graph of an object P moving in a straight line from a fixed point O for seconds.



Use this diagram to estimate:

a) the initial displacement of P

m ✓ estimate of 45 m ± 1 m

b) when the object P is at rest

m/s

when seconds ✓ at least two of t = 1, 6 and 10 seconds ± 0.1 seconds

✓ all three times

c) the velocity of P at seconds

m/s ✓ estimate of 5 m/s ± 1 m/s

d) when the velocity of P is -12 m/s.

m/s

when seconds ✓ estimate of t = 3 seconds ± 0.5 seconds

**Bunbury Senior High School Name:\_\_SOLUTIONS\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Mathematics: Methods Unit 2**

**Test 6, 2018**

**Differentiation and Integration with Applications**

**Section Two:**

**Calculator-assumed**

Time allowed for this section: 35 minutes

Total marks for this section: 32 marks

**Materials required/recommended for this section:**

SCSA Formula Sheet

Notes on one sheet (both sides) of A4 paper

Up to three approved calculators

**Instructions to candidates**

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

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| **Question 6** | **[4 marks]** |

Find the derivative of from first principles.

✓ correctly applies function into first principles limit

✓ expands

✓ collects like terms and factorises out *h*

✓ evaluates the limit (must have reasoning)

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| **Question 7** | **[3 marks]** |

The curve has a stationary point at . Find the value of .

✓ correct derivative

✓ substitutes and solves

✓ correct

|  |  |
| --- | --- |
| **Question 8** | **[6 marks]** |

Use derivatives to find the stationary points(s), for the curve, . Hence, sketch the curve, clearly indicating all stationary point(s) and intercept(s).

✓ correct derivative

or ✓ solves

stationary points at and ✓ determines stationary points

-intercept at

solve for roots

and

roots at and ✓ determines second -intercept

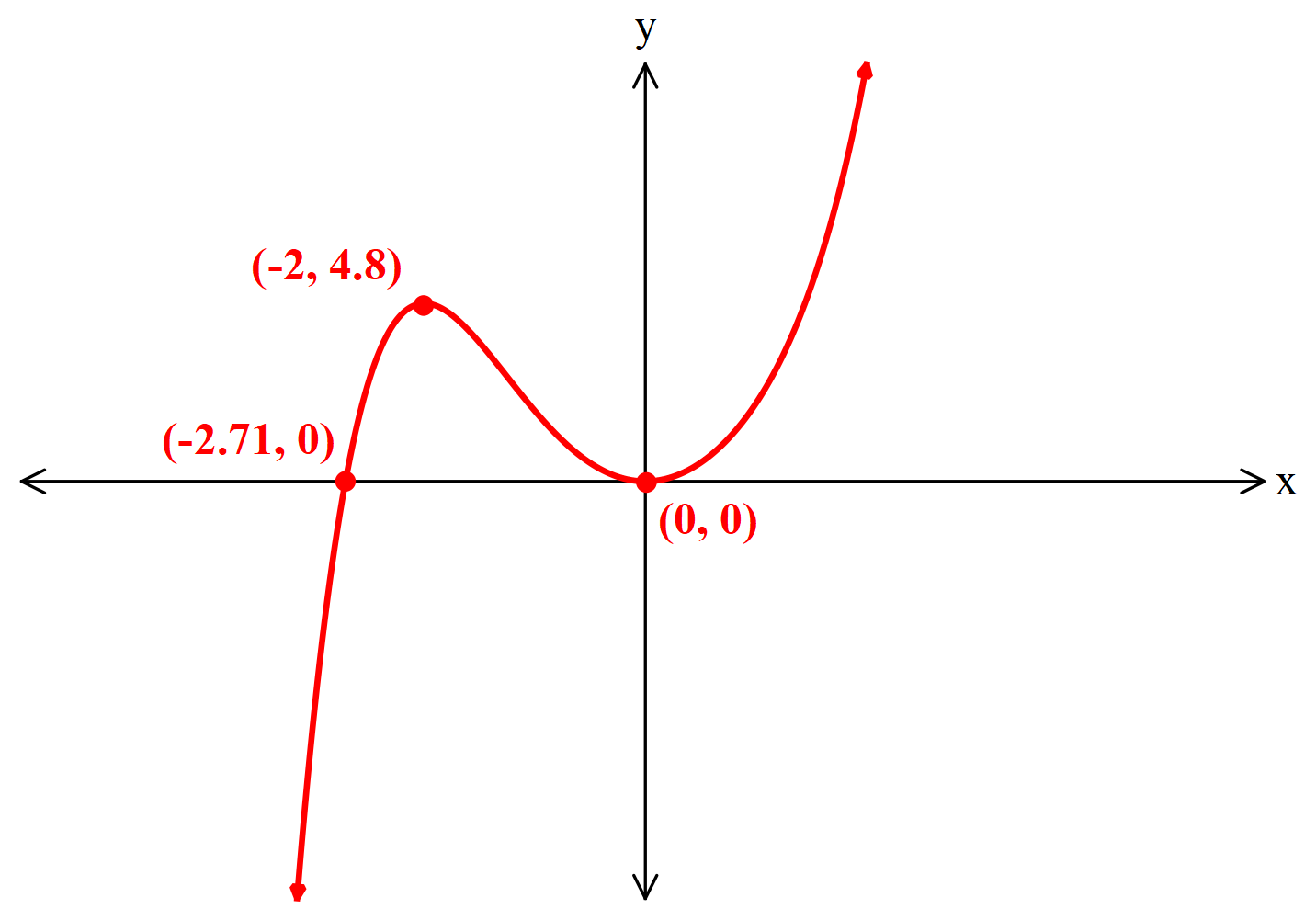
considering the leading term

when and

when

✓ correct shape of function when sketched

✓ indicates stationary points and intercepts on sketch



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| **Question 9** | **[1, 2, 2 = 5 marks]** |

A car is travelling in a straight line. The car’s distance metres, from a fixed point O, is given by,

for , where time is in seconds.

a) Find the initial velocity of the car.

ms-1 ✓ correct initial velocity

b) Find when the velocity of the car is 18 ms-1.

✓ uses first derivative of displacement to determine the velocity function

and substitutes for 18

when seconds ✓ correct, positive solution only since

c) Find the average speed of the car in the first 4 seconds.

m ✓ determines displacement after 4 seconds

ms-1

ms-1 ✓ correct average speed

|  |  |
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| **Question 10** | **[1, 1, 2, 2 = 6 marks]** |

A crack in an oil pipe on the ocean floor leaks oil to the surface. The total area of oil contamination on the ocean’s surface, in square kilometres (km2), is modelled by the function, , where is the number of days since the pipe started to leak oil.

a) Find an expression for the rate of change in with respect to .

or

✓ correct derivative

b) Find the rate of change in with respect to after 1 day of the oil starting to leak into the ocean.

km2/day ✓ correct value

The oil company that owns the pipe are quick to respond to the broken pipe and start to repair the pipe and clean the ocean surface of oil contamination.

c) After how many day is the spread of oil on the ocean surface stopped?

solve

days ✓ solves and obtains correct value for

after 3 days the spread of oil has stopped ✓ after 3 days

d) Find the rate of change in with respect to after 6 days of the oil starting to leak into the ocean. Briefly

comment on the meaning of this result.

km2/day ✓ correct value

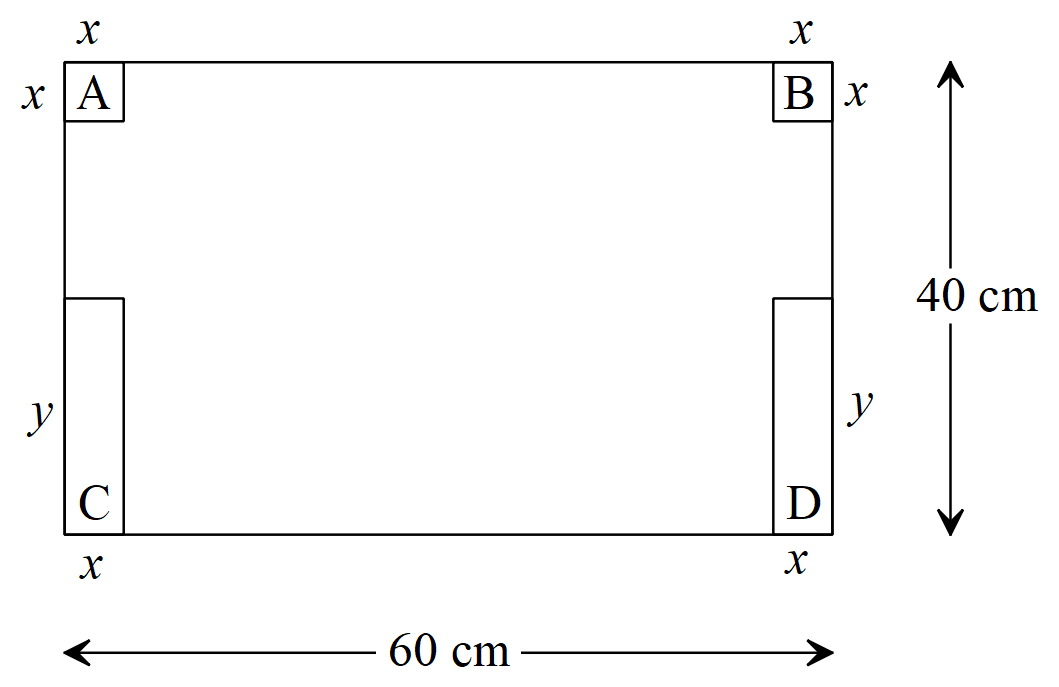
The negative rate means the area is getting smaller, that is, the repairs and clean-up are both reducing the area contaminated by oil.

✓ indicates that the negative rate means a reducing area

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| **Question 11** | **[1, 3, 4 = 8 marks]** |

A rectangular sheet of cardboard, 40 cm by 60 cm, is to be made into a closed rectangular box. A square of side,

cm, is removed from each of the corners A and B of the cardboard. A rectangle cm by cm, is removed from each of the corners C and D of the cardboard.



a) Show that the length of the box is given by .

✓ shows that the 60 cm side of the cardboard sheet

makes the box length plus

b) Show that the volume, , of the box is given by

Let the width of the box

However,

✓ shows that the 40 cm side of the cardboard sheet makes twice the box width plus

✓ determines the expression for width in terms of

The height of the box will be given by

i.e. ✓ indicates the height of the box is length

since

c) Use calculus to find the dimensions of the box that will maximise its volume, all correct to 2 decimal places.

✓ determines

solve

cm or cm ✓ solves for height

✓ discounts the solution for the height as the corresponding width will be negative

(either formally as a statement or informally by not including it as a solution)

cm cm cm

✓ states all three dimensions correctly